Omnidirectional Multisensory Perception Fusion for Long-Term Place Recognition
Supplementary Material

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I. Proof of Theorem 1

In the following, we prove that Algorithm 1 in the main paper decreases the value of the objective function with each iteration and converges to the optimal value. But first, we present a lemma:

***Lemma** 1:* For any two given vectors $\mathbf{v}$ and $\tilde{\mathbf{v}}$, the following inequality relation holds: $\|\tilde{\mathbf{v}}\|_2 - \frac{\|\tilde{\mathbf{v}}\|_2^2}{2\|\mathbf{v}\|_2} \leq \|\mathbf{v}\|_2 - \frac{\|\mathbf{v}\|_2^2}{2\|\mathbf{v}\|_2}$.

Proof: We have:

$$\|\widetilde{\mathbf{v}}\|_2 - \|\mathbf{v}\|_2)^2 \leq 0 \quad (1)$$

$$-\|\widetilde{\mathbf{v}}\|_2^2 - \|\mathbf{v}\|_2^2 + 2\|\widetilde{\mathbf{v}}\|_2\|\mathbf{v}\|_2 \leq 0 \quad (2)$$

$$2\|\widetilde{\mathbf{v}}\|_2\|\mathbf{v}\|_2 - \|\widetilde{\mathbf{v}}\|_2^2 \leq \|\mathbf{v}\|_2^2 \quad (3)$$

$$\|\widetilde{\mathbf{v}}\|_2 - \frac{\|\widetilde{\mathbf{v}}\|_2^2}{2\|\mathbf{v}\|_2} \leq \|\mathbf{v}\|_2 - \frac{\|\mathbf{v}\|_2^2}{2\|\mathbf{v}\|_2} \quad (4)$$

■

From Lemma 1, we can derive the following corollary:

***Corollary** 1:* For any two given matrices $\mathbf{M}$ and $\tilde{\mathbf{M}}$, the following inequality relation holds: $\|\tilde{\mathbf{M}}\|_C - \frac{\|\tilde{\mathbf{M}}\|_C^2}{2\|\mathbf{M}\|_C} \leq \|\mathbf{M}\|_C - \frac{\|\mathbf{M}\|_C^2}{2\|\mathbf{M}\|_C}$.

***Theorem** 1:* Algorithm 1 converges to the optimal solution to the optimization problem in Eq. (4) (of main paper).

Proof: According to Steps 3 of Algorithm 1, we know

$$\mathbf{W}(t+1) = \min_{\mathbf{W}} \|\mathbf{R}\mathbf{X}^\top\mathbf{W} - \mathbf{Y}\|_F^2 \quad (5)$$
$$+\gamma_1 Tr\mathbf{W}^\top\mathbf{D}(t+1)\mathbf{W} + \gamma_2 Tr\mathbf{W}^\top\tilde{\mathbf{D}}(t+1)\mathbf{W}.$$

Then, we can derive that

$$\begin{aligned}
&\mathcal{J}(t+1) + \gamma_1 Tr\mathbf{W}^\top(t+1)\mathbf{D}(t+1)\mathbf{W}(t+1) \\
&+\gamma_2 Tr\mathbf{W}^\top(t+1)\tilde{\mathbf{D}}(t+1)\mathbf{W}(t+1) \\
\leq\ &\mathcal{J}(t) + \gamma_1 Tr\mathbf{W}^\top(t)\mathbf{D}(t+1)\mathbf{W}(t) \\
&+\gamma_2 Tr\mathbf{W}^\top(t)\tilde{\mathbf{D}}(t+1)\mathbf{W}(t),
\end{aligned}$$

where $\mathcal{J}(t) = \|\mathbf{R}\mathbf{X}^\top\mathbf{W}(t) - \mathbf{Y}\|_F^2$.
After substituting the definition of $\mathbf{D}$ and $\tilde{\mathbf{D}}$, we obtain

$$\begin{aligned}
&\mathcal{J}(t+1) + \gamma_1 \frac{\|\mathbf{W}(t+1)\|_C^2}{2\|\mathbf{W}(t)\|_C} \quad (6)\\
&+\gamma_2 \sum_{i=1}^m \frac{\|\mathbf{w}^i(t+1)\|_2^2}{2\|\mathbf{w}^i(t)\|_2} \\
\leq\ &\mathcal{J}(t) + \gamma_1 \frac{\|\mathbf{W}(t)\|_C^2}{2\|\mathbf{W}(t)\|_C} + \gamma_2 \sum_{i=1}^m \frac{\|\mathbf{w}_i(t)\|_2^2}{2\|\mathbf{w}^i(t)\|_2}.
\end{aligned}$$

From Lemma 1 and Corollary 1, we can derive that

$$\begin{aligned}
&\|\mathbf{W}(t+1)\|_C - \frac{\|\mathbf{W}(t+1)\|_C^2}{2\|\mathbf{W}(t)\|_C} \leq \\
&\|\mathbf{W}(t)\|_C - \frac{\|\mathbf{W}(t)\|_C^2}{2\|\mathbf{W}(t)\|_C}. \quad (7)
\end{aligned}$$

and,

$$\begin{aligned}
&\sum_{i=1}^m \|\mathbf{w}^i(t+1)\|_2 - \sum_{i=1}^m \frac{\|\mathbf{w}^i(t+1)\|_2^2}{2\|\mathbf{w}^i(t)\|_2} \leq \\
&\sum_{i=1}^m \|\mathbf{w}^i(t)\|_2 - \sum_{i=1}^m \frac{\|\mathbf{w}^i(t)\|_2^2}{2\|\mathbf{w}^i(t)\|_2}, \quad (8)
\end{aligned}$$

Adding Eq. (6)-(8) on both sides, we have

$$\begin{aligned}
&\mathcal{J}(t+1) + \gamma_1\|\mathbf{W}(t+1)\|_C \quad (9)\\
&+\gamma_2 \sum_{i=1}^m \|\mathbf{w}^i(t+1)\|_2 \\
\leq\ &\mathcal{J}(t) + \gamma_1\|\mathbf{W}(t)\|_C + \gamma_2 \sum_{i=1}^m \|\mathbf{w}^i(t)\|_2.
\end{aligned}$$

Eq. (9) shows the value of the objective function decreases in each iteration. Because our objective function is convex, Algorithm 1 converges to the optimal value. ■

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